

CHAPTER 23 · GENERAL NAVIGATION

The Polar Stereographic Chart

Geometric projection · Scale $\sec^2(1/2 \text{ co-lat}) \cdot n = 1$ · Straight-line track problems

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Introduction — A True Perspective Projection

Of all the chart projections studied in this course, the **Polar Stereographic** is the only *true geometric (perspective) projection*. It is constructed by placing a flat sheet of paper touching the **North Pole** (the point of tangency), with the light source at the **South Pole** (diametrically opposite).

The graticule is cast by straight light paths from the South Pole through the globe and onto the flat sheet, producing the characteristic circular parallel / radial meridian graticule.

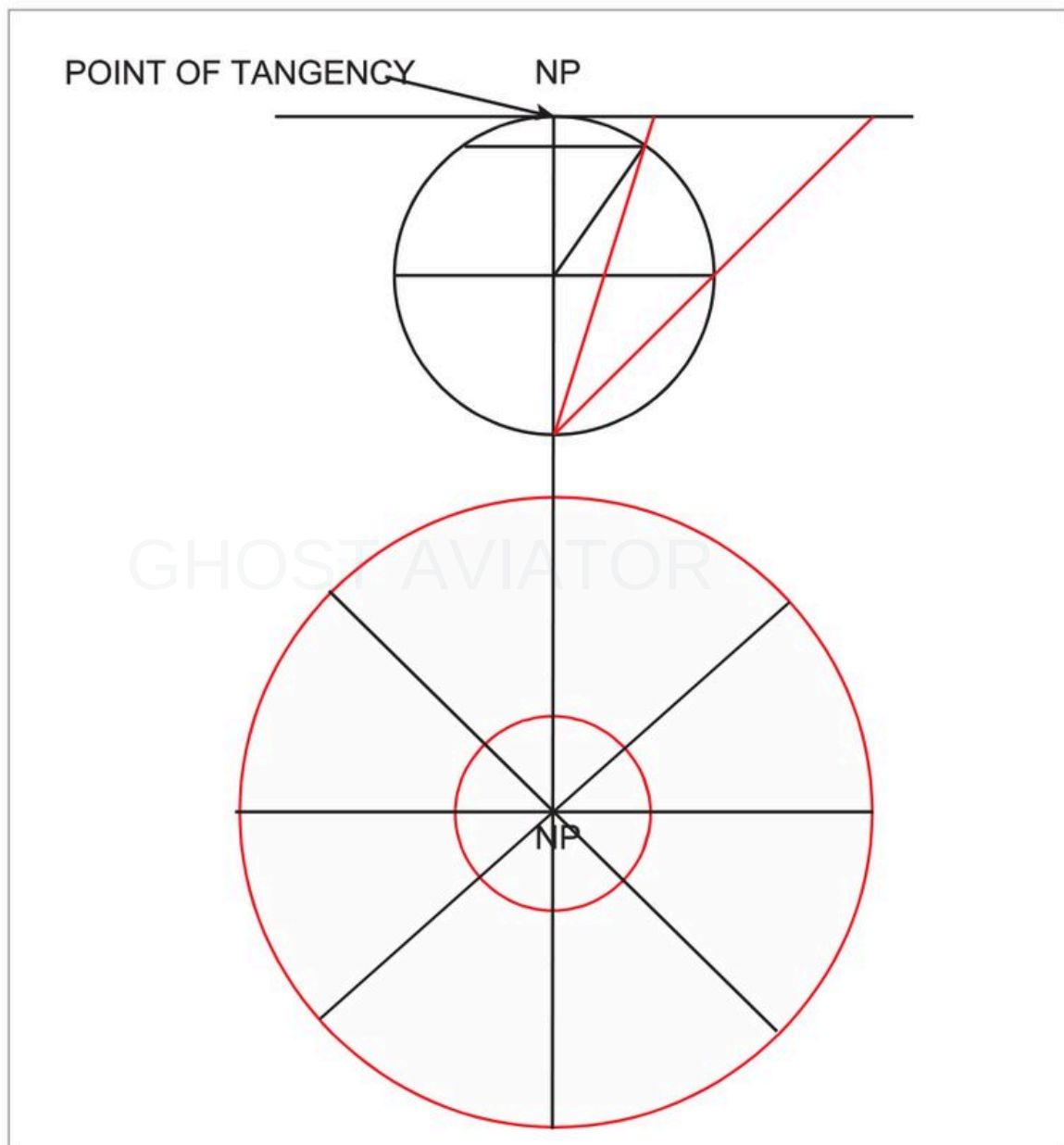


Figure 334 — A Polar stereographic projection

Figure 1 — Polar Stereographic projection construction: flat sheet touches the North Pole; light source at South Pole; the graticule is projected geometrically outward from the pole

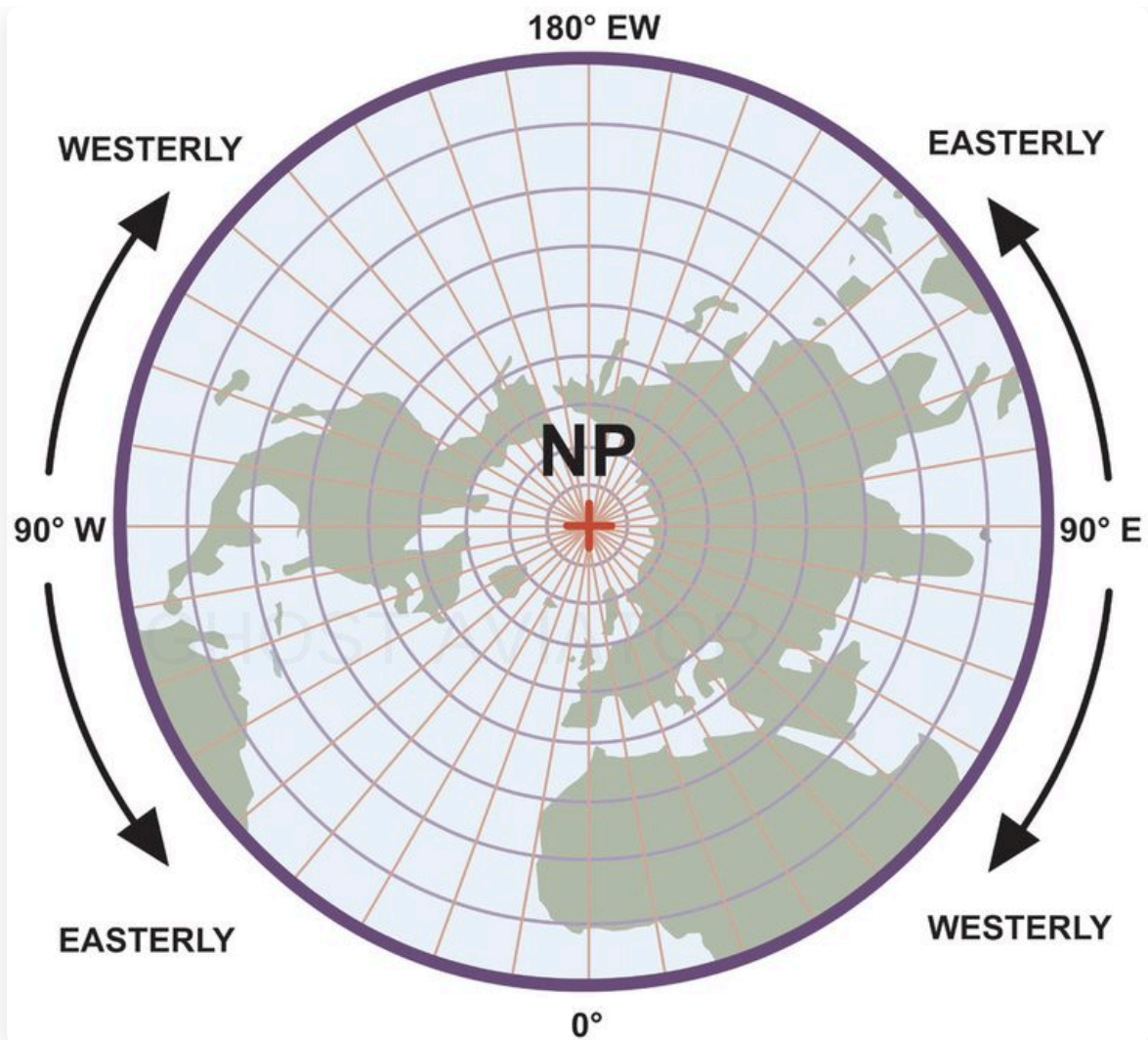


Figure 2 — Polar Stereographic graticule: meridians are straight lines radiating from the pole; parallels are concentric circles; the spacing between parallels increases with distance from the pole

The Co-latitude

The co-latitude is a key concept for polar chart calculations:

$$\text{Co-latitude} = 90^\circ - \text{latitude}$$

It measures the angular distance from the pole (rather than from the Equator)

Examples:

- Latitude 90°N (the North Pole): co-latitude = 0°
- Latitude 78°N: co-latitude = 12°
- Latitude 70°N: co-latitude = 20°
- Latitude 60°N: co-latitude = 30°
- Latitude 0° (Equator): co-latitude = 90°

The co-latitude is used because polar chart properties (scale expansion, distance from the pole) are measured from the pole rather than the Equator.

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Scale Expansion

Scale is **correct at the Pole** (the point of tangency). Elsewhere it expands with increasing co-latitude (i.e. with decreasing latitude / distance from the pole):

Scale at latitude ϕ = Scale at Pole $\times \sec^2(\frac{1}{2}$ co-latitude)

Where: co-latitude = $90^\circ - \phi$; $\sec(\theta) = 1/\cos(\theta)$

Worked Example — Scale at 78°N

If scale at Pole = 1:1 000 000:

Co-latitude of 78°N = $90^\circ - 78^\circ = 12^\circ$. Half co-lat = 6° .

Scale at 78°N = $(1/1\,000\,000) \times \sec^2(6^\circ) = (1/1\,000\,000) \times (1/\cos 6^\circ)^2 = (1/1\,000\,000) \times (1/0.9945)^2$

= $(1/1\,000\,000) \times (1.00554)^2 \approx (1/1\,000\,000) \times 1.011 = \mathbf{1/989\,074}$

→ Scale change from 90°N to 78°N: less than 1%. Chart is within 1% of correct scale.

Scale Zones

The near-constant scale near the pole makes this chart ideal for polar flying:

- **90° to 78°N:** Scale within 1% of correct (Pole scale)
- **78° to 70°N:** Scale within 3% of correct
- **Below 70°N:** Scale error exceeds 3%; the chart becomes less suitable

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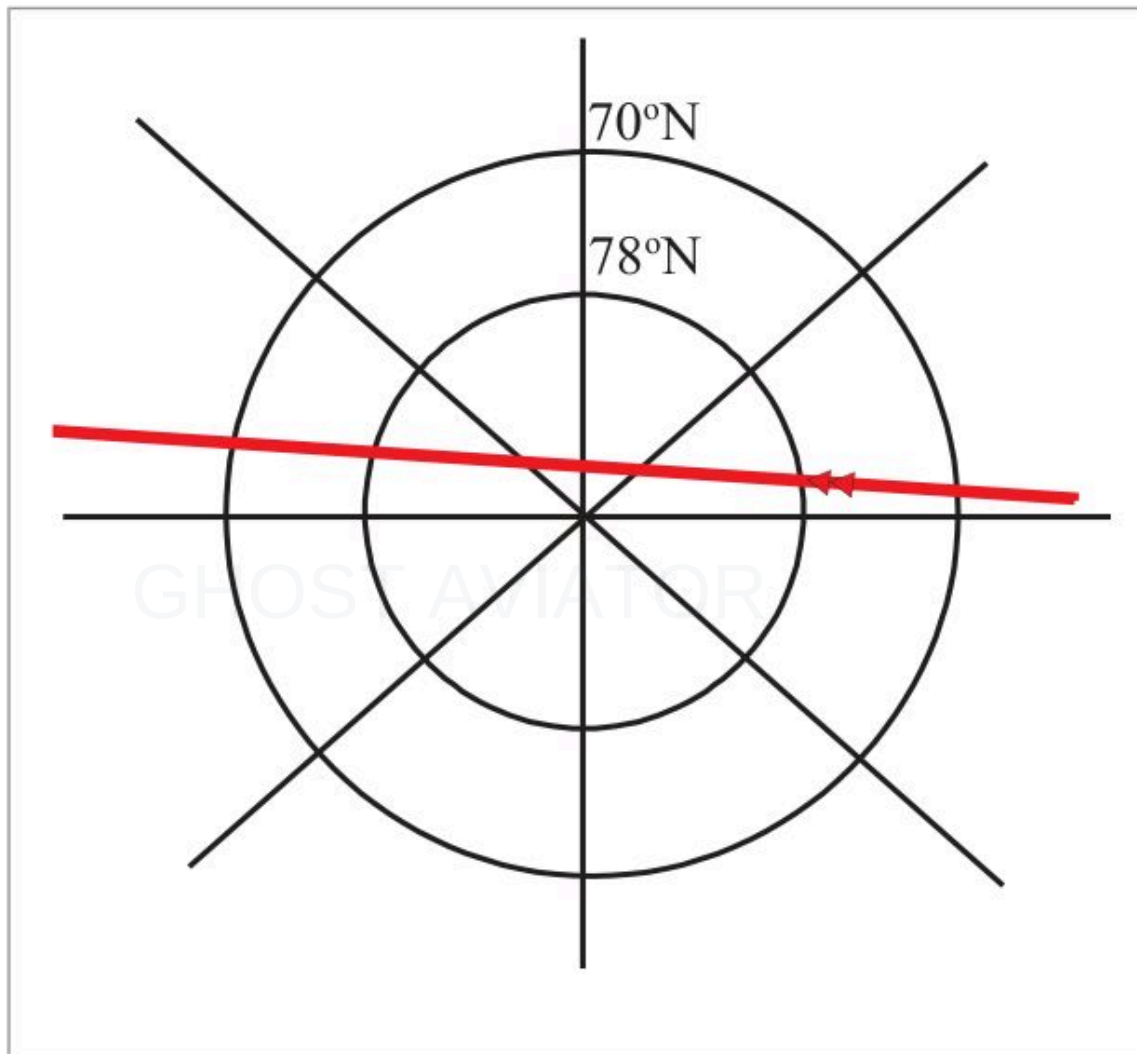


Figure 22.2 Track near Pole

Figure 3 — Scale zones on the Polar Stereographic: the 78°N parallel marks the 1% boundary and the 70°N parallel marks the 3% boundary — this 1% zone covers about 720 NM radius from the pole

The practical significance: the 1% scale zone extends from the Pole to 78°N. The distance from Pole to 78°N = $12^\circ \times 60 \text{ NM} = \mathbf{720 \text{ NM}}$. An aircraft crossing the polar region (entering on one side, exiting the other) travels about 1 400 NM in the 1% zone — approximately 3 hours at typical jet speeds. A chart useful for 3 hours of polar flying with minimal scale error is extremely practical.

Chart Properties

Property	Detail
Projection type	Azimuthal (plane), geometric (perspective), orthomorphic
Orthomorphism	Yes — meridians radiate from pole, parallels are concentric circles; expansion is equal in all directions from any point, satisfying both orthomorphism conditions
Scale	Correct at Pole. Expands as $\sec^2(\frac{1}{2} \text{ co-lat})$. Within 1% from 90°N to 78°N; within 3% from 78°N to 70°N
Graticule	Meridians: straight lines from pole. Parallels: concentric circles centred at pole (uneven spacing — wider apart with distance from pole)
Shape distortion	Shapes and areas become increasingly distorted away from the pole
Practical use	Polar route planning and plotting; ETOPS operations over polar areas; some meteorological charts

Chart Convergence — $n = 1$

On the Polar Stereographic chart, the meridians converge at the pole at exactly the same rate as they do on the Earth's surface. This gives the chart a **convergence factor $n = 1$** :

Chart Convergence = change of longitude $\times 1 =$ **ch.long**

This is CONSTANT across the whole chart and CORRECT at the Pole only

On the real Earth, convergency = $\text{ch.long} \times \sin(\text{latitude})$. At the Pole, $\sin(90^\circ) = 1$, so Earth convergency = ch.long . Chart convergence (always = ch.long) equals Earth convergency only at the Pole. Elsewhere, chart convergence $>$ Earth convergency (since $\sin(\text{lat}) < 1$ at any latitude other than the Pole).

N = 1: THE KEY NUMBER FOR POLAR STEREO PROBLEMS

For any two meridians on a Polar Stereographic chart, the angle *between* them on the chart = their difference in longitude. This is the starting point for all straight-line track problems on this projection.

Rhumb Lines and Great Circles

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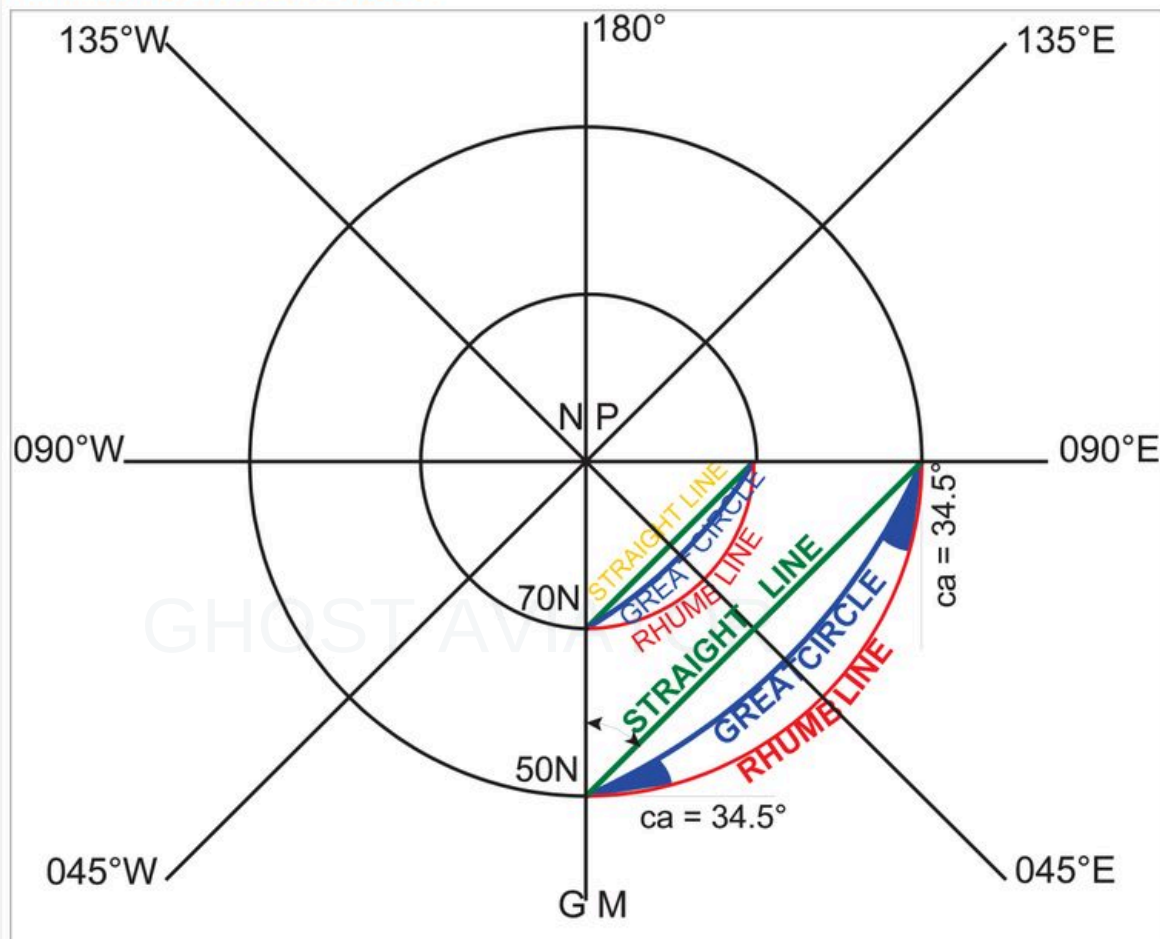


Figure 33.4 — Great circle, rhumb lines and straight lines

Figure 4 — Great circles, rhumb lines and straight lines on a Polar Stereographic chart between two meridians (000°E and 090°E) at 50°N and 70°N — the straight line on the chart sits between the RL and GC in terms of curvature

Rhumb Lines

Rhumb lines are curves **concave to the Pole** (same as on the Lambert chart, but here the "Parallel of Origin" is the pole itself). The curvature is considerable.

Great Circles

Great circles are also curves **concave to the Pole**, but with **less curvature than Rhumb Lines** in the same hemisphere.

Straight Lines vs Great Circles at Different Latitudes

The difference between a straight line on the chart and the corresponding Great Circle can be calculated:

- **At 50°N, 90° ch.long:** Angle between straight line and RL = $\frac{1}{2} \times \text{chart convergence} = \frac{1}{2} \times 90^\circ = 45^\circ$. Conversion angle = $\frac{1}{2} \times 90 \times \sin(50^\circ) = 34.5^\circ$.
Difference = $45^\circ - 34.5^\circ = 10.5^\circ$. Too large to treat as equal.
- **At 70°N, 90° ch.long:** $\frac{1}{2} \text{ CC} = 45^\circ$. Conversion angle = $\frac{1}{2} \times 90 \times \sin(70^\circ) = 42.3^\circ$.
Difference = $45^\circ - 42.3^\circ = 2.7^\circ$. Acceptably small.

RULE OF THUMB

At latitudes above **70°N** (or 70°S), a straight line on a Polar Stereographic chart may be taken as a Great Circle. Below 70°N, the approximation breaks down.

Straight-line Track Problems

These are the most common exam questions on this chapter. Two methods work:

- **Triangle geometry:** Draw the triangle formed by the Pole, point A, and point B. Use the fact that the angle at the Pole = ch.long (since $n = 1$). Solve the triangle (isosceles if A and B are at the same latitude).
- **Rhumb Line method:** Find the RL track (easy — same latitude means 090° or 270°). Then subtract half chart convergence to find the initial straight-line track.

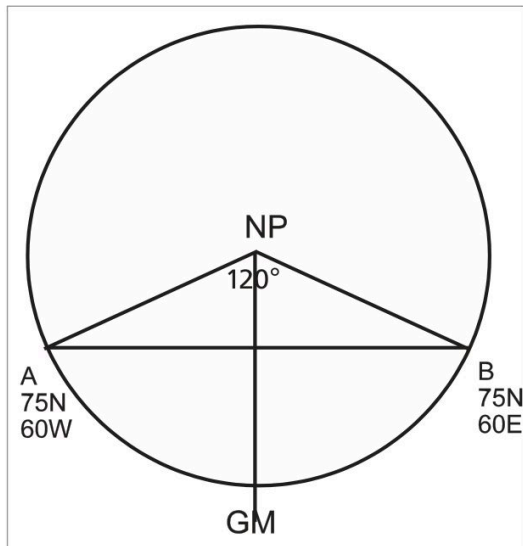


Figure 5 — Example 1 diagram — isosceles triangle A(75N 60W)–Pole–B(75N 60E); apex angle = 120° (ch.long); base angles each = 30°

or way of solving this problem by use of the Rhumb Line method. If A is at 75N then the Rhumb Line track between them is the parallel of latitude and it has a Rhumb Line track of $090^\circ(T)$.

Convergence between 60W and 60E is 120° . Therefore half chart convergence is 60° . The difference between the Rhumb Line and the straight line. Therefore the Rhumb Line track of $090^\circ(T)$ gives us an initial straight-line track

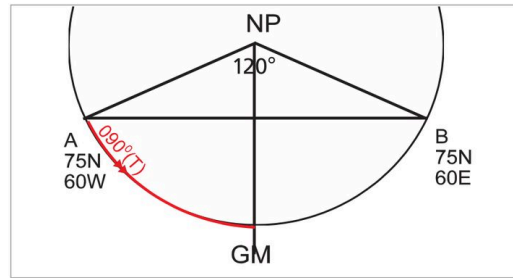


Figure 6 — Example 1 using the RL method —
RL track = $090^\circ(T)$; half chart convergence = 60° ; straight-line track at A = $090^\circ - 60^\circ = 030^\circ(T)$

Example 1: Initial straight-line track from A (75N 60W) to B (75N 60E)

Method 1 — Triangle: A and B are both at $75^\circ N$ (co-lat = 15° each). Angle at Pole = ch.long = 60W to 60E = 120° . Isosceles triangle: two equal co-lat sides of 15° . Internal angles sum to 180° : angles at A and B = $(180^\circ - 120^\circ)/2 = 30^\circ$ each.

Direction from A to North Pole = $000^\circ(T)$. Straight-line to B is 30° to the right = **$030^\circ(T)$** .

Method 2 — RL: A and B are both at $75^\circ N \rightarrow$ RL track = $090^\circ(T)$. Chart convergence = 120° . Half CC = 60° . Subtract: $090^\circ - 60^\circ =$ **$030^\circ(T)$** ✓

Initial straight-line track A $\rightarrow$ B = $030^\circ(T)$

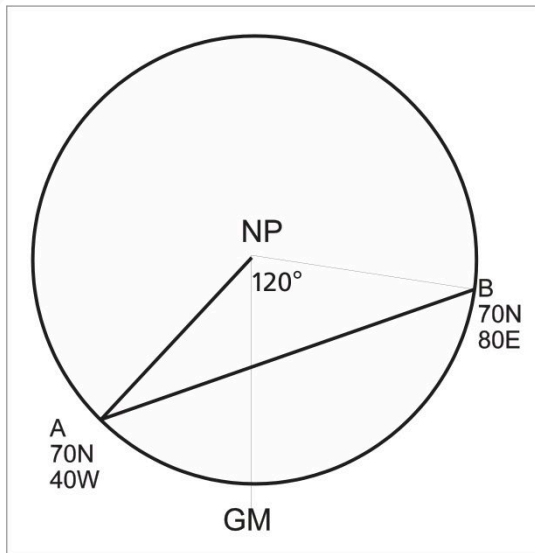


Figure 33.7 — Polar stereographic situation

Figure 7 — Example 2 — vertex longitude: the highest latitude on a straight line between two points occurs at the shortest distance from the Pole, where the perpendicular from the Pole meets the line

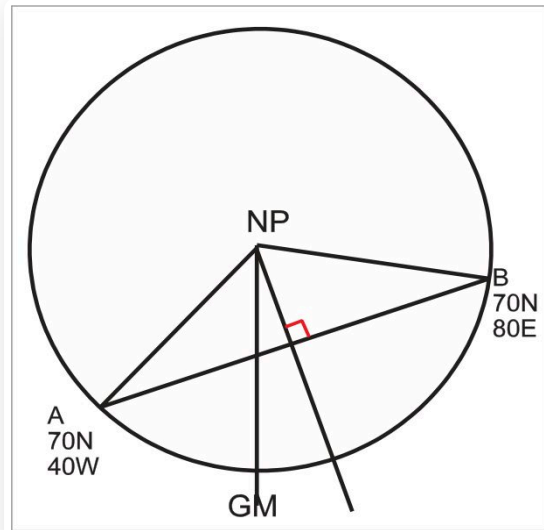


Figure 33.8 — Right angles to track

Figure 8 — Example 2 — perpendicular from Pole to track meets at mid-meridian (020°E = mid-point of 40°W and 80°E)

Example 2: Highest latitude on straight-line from A (70°N 40°W) to B (70°N 80°E)

A and B are both at 70°N . Pole–A–B is an isosceles triangle.

The highest latitude (shortest distance from Pole) is where the line is closest to the Pole. By symmetry, this must be at the **perpendicular bisector** from the Pole.

Mid-meridian between 40°W and 80°E = $(-40 + 80)/2 = 20^{\circ}\text{E} \rightarrow$
 020°E .

Highest latitude at longitude 020°E

Figure 9: Right-angled triangle.

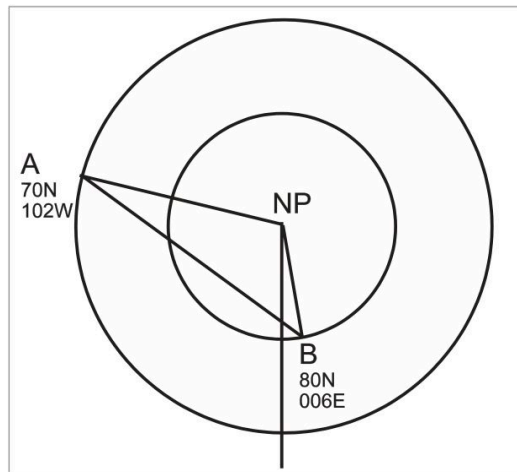


Figure 23.9

Figure 9 — Example 3 setup — A (70N 102W) and B (80N 006E) at different latitudes; highest latitude along the line is at 035°W

Right-angled triangle, between A, the North Pole, and the point c. The difference between 035W and 102W, so it is 67°. Angle c is the angle at the North Pole. The angle must add up to 180°. So angle a must be 23°.

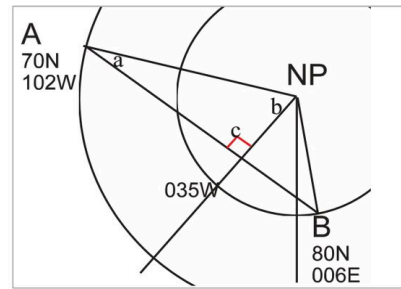


Figure 23.10

Figure 10 — Example 3 solution — the perpendicular from the Pole to the highest-latitude point creates a right-angled triangle; angle $b = 102^\circ\text{W} - 035^\circ\text{W} = 67^\circ$; angle $a = 180^\circ - 90^\circ - 67^\circ = 23^\circ$

Example 3: Initial track from A (70N 102W) to B (80N 006E) — highest latitude at 035°W

A and B are at *different* latitudes, so there's no isosceles triangle. Use the highest-latitude (vertex) information instead.

Draw the right-angled triangle: Pole $\rightarrow$ vertex (at 035°W) $\perp$ to the straight line. The angle at the Pole (angle b) = difference between A's longitude (102°W) and vertex (035°W) = **67°**.

Angle at vertex = 90° (perpendicular by definition). Angle at A = $180^\circ - 90^\circ - 67^\circ = \mathbf{23^\circ}$.

Direction from A to Pole = 000°(T). The 23° is measured to the RIGHT of north (east of north, toward B). Initial straight-line track = **023°(T)**.

Initial straight-line track from A = 023°(T)

Summary — Properties of a Polar Stereographic Chart

Projection type	Azimuthal (flat sheet), geometric/perspective, orthomorphic
Scale	Correct at Pole. Elsewhere expands as $\sec^2(1/2 \text{ co-lat})$. Within 1%: $90^\circ \rightarrow 78^\circ$. Within 3%: $78^\circ \rightarrow 70^\circ$.
Graticule	Meridians: straight lines from Pole. Parallels: concentric circles (uneven — increasingly spaced).
Orthomorphic	Yes — equal scale expansion in all directions at any point.
Chart Convergence	= ch.long. Constant everywhere. Correct only at the Pole. $n = 1$.
Rhumb Lines	Curves concave to the Pole (except meridians).
Great Circles	Curves concave to the Pole, but with less curvature than RLs. Treated as straight lines above 70° .
Straight-line track	Initial track at A = $000^\circ(\text{T to Pole}) \pm \text{angle from isosceles/right-angle triangle}$. Or: $\text{RL} \pm \text{half CC}$.

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