

CHAPTER 21 · GENERAL NAVIGATION

Lambert's Conformal Chart

Simple conic → Lambert's modification · Standard parallels · Chart convergence · Great circles as near-straight lines

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Why Conical Charts?

The Mercator chart has two important limitations for modern aviation:

- **Great Circles are not straight lines** — they appear as curves concave to the Equator. Modern aircraft with FMS/INS fly Great Circle tracks, and a chart where the planned track appears curved is inconvenient.
- **Scale is not constant** — scale changes significantly with latitude on a Mercator chart, making it impossible to measure distances with a ruler. Instead, dividers must be used, adjusted to different sizes at different latitudes.

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After 1960, with the advent of automatic computing (INS, IRS, FMS, GPS), it became practical to fly **Great Circle tracks**. A chart on which a Great Circle appears as a straight line is therefore far more useful for route planning.

The solution: the **Lambert's Conical Conformal Chart**, developed in 1777 by Swiss mathematician **Johann Heinrich Lambert**.

The Simple Conical Projection

The basis of the Lambert chart is the simple conic. A light source at the centre of the Reduced Earth casts shadows of the graticule onto the inside surface of a cone placed over the globe. The cone touches the globe at the **parallel of tangency**. The cone is then slit and opened flat.

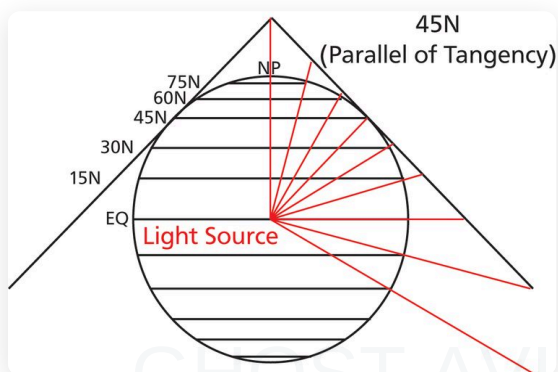


Figure 1 — Principle of simple conical projection — light source at the centre projects the graticule onto the cone; scale is correct at the parallel of tangency

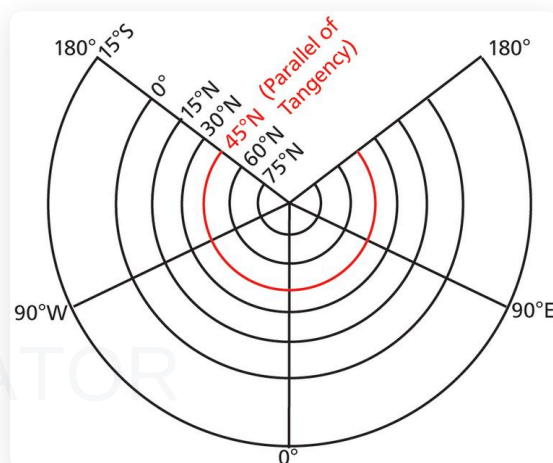


Figure 2 — Simple conical graticule — when the cone is slit and opened flat, 360° of longitude appears as a sector of a circle; meridians radiate from the pole; parallels are arcs of concentric circles

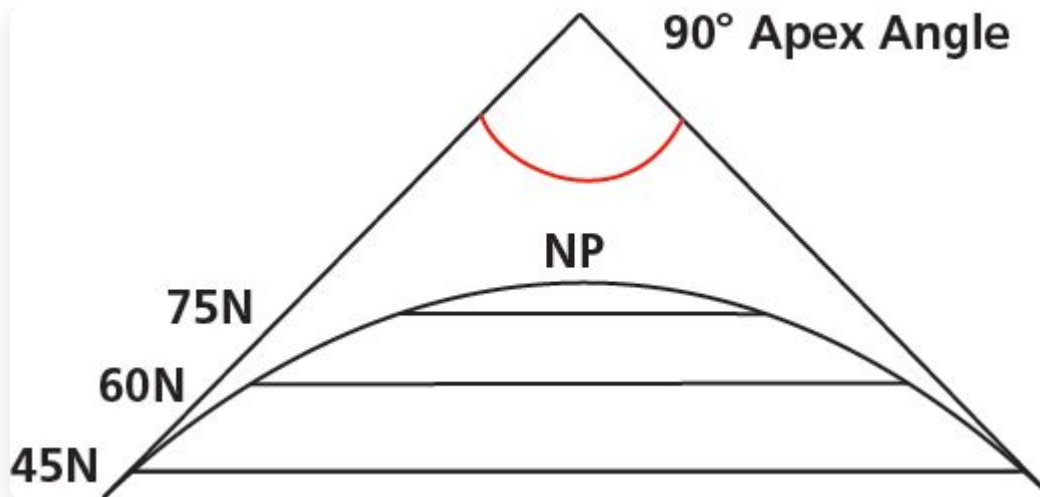


Figure 3 — Apex angle is twice the parallel of tangency: at 45°N tangency, the apex angle = 90° ; at 60°N , apex = 120° ; at 90°N (flat sheet), apex = 180°

The Arc of Sector Formula

When the cone is opened flat, 360° of longitude is represented by an arc of *less than* 360° . The size of this arc depends on the parallel of tangency:

Arc of sector = change of longitude $\times$ sin(parallel of tangency)

For a full 360° : arc = $360^\circ \times \sin(\text{parallel of tangency})$

Examples:

- Parallel of tangency 45°N : arc = $360^\circ \times \sin(45^\circ) = 360^\circ \times 0.707 = \mathbf{255^\circ}$.
Missing sector = 105° .
- Parallel of tangency 60°N : arc = $360^\circ \times \sin(60^\circ) = 360^\circ \times 0.866 = \mathbf{312^\circ}$.
Missing sector = 48° .
- Parallel of tangency 90°N : arc = $360^\circ \times \sin(90^\circ) = 360^\circ \times 1.0 = \mathbf{360^\circ}$. No missing sector (flat sheet — the Polar Stereographic case).

The relationship between 360° of longitude and the arc of the sector is called the **constant of the cone**, denoted **n**:

$$n = \sin(\text{parallel of tangency}) = \text{angle between meridians} / \text{ch.long}$$

Also called the convergence factor — used in chart convergence calculations

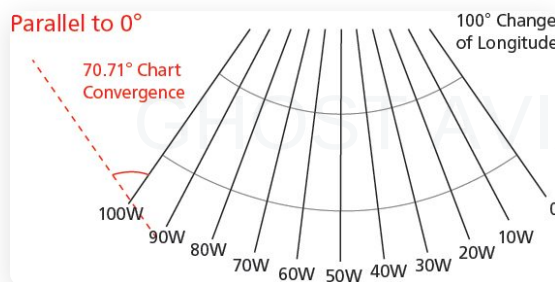


Figure 4 — Change of longitude vs chart convergence: 100° ch.long is represented by a smaller angle on the chart — the ratio depends on n (sin of parallel of tangency)

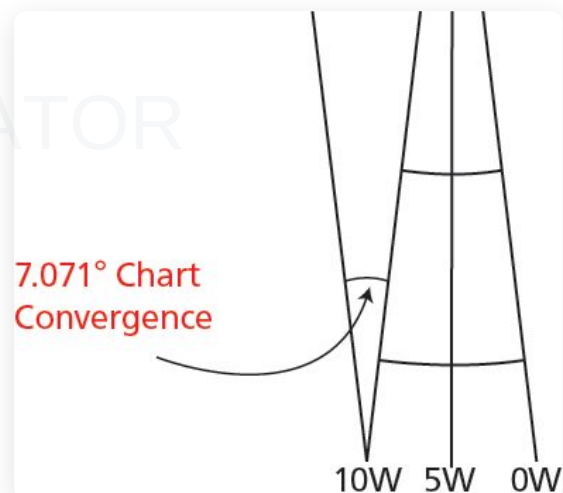


Figure 5 — Chart and Earth meridian angles compared — on the chart the angle between meridians is less than the Earth angle between the same meridians

Scale on the Simple Conic Projection

On the simple conic, scale is **correct only at the parallel of tangency**. It expands away from this parallel in both directions. The expansion is quite rapid — much faster than on a Lambert chart.

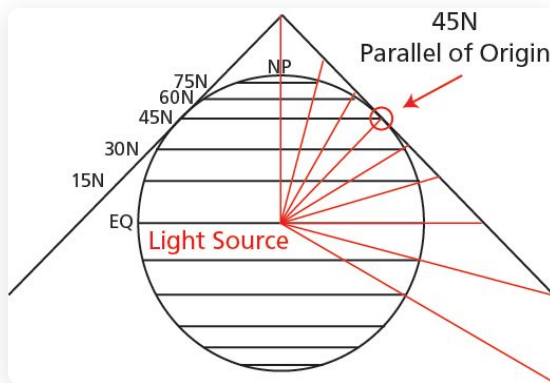


Figure 6 — Scale on the simple conic: correct only at the parallel of tangency (shown in green); expands away from it toward higher and lower latitudes

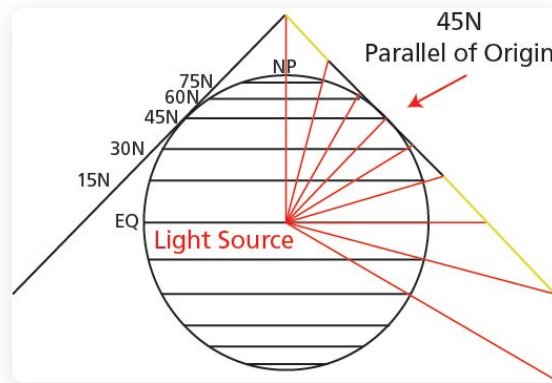
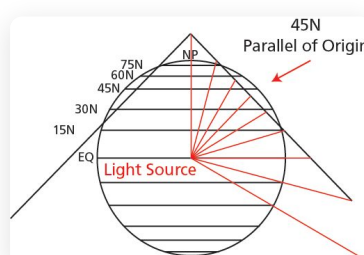


Figure 7 — Illustrating scale expansion on the simple conic — the expanding spacing of the parallels shows how scale grows away from the tangent parallel

Lambert's Modification — Bringing the Cone Inside

Lambert's key insight: by **pushing the cone inside the Reduced Earth** (rather than leaving it touching the outside), the single line of correct scale at the parallel of tangency is replaced by **two parallels of correct scale** — the **standard parallels**. Between the standard parallels, scale is slightly compressed; outside them, scale is slightly expanded.

The result is a chart with near-constant scale across the whole projection, making distance measurement with a ruler practical.



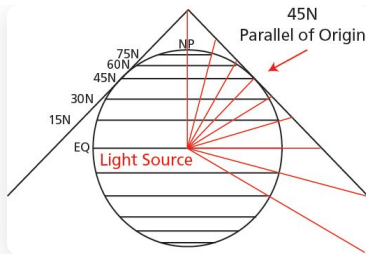


Figure 8 — Lambert step 1: simple conic showing scale correct at tangent parallel, expanding rapidly outside

Figure 9 — Lambert step 2: cone pushed inside the RE — the single tangent parallel becomes two secant parallels (standard parallels)

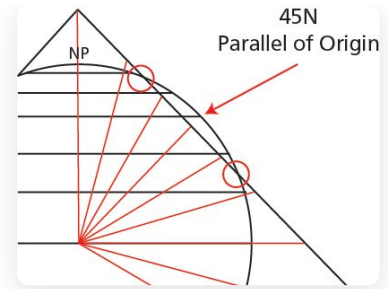


Figure 10 — Lambert modification — the cone now intersects the Reduced Earth at two latitudes (standard parallels) rather than touching at one

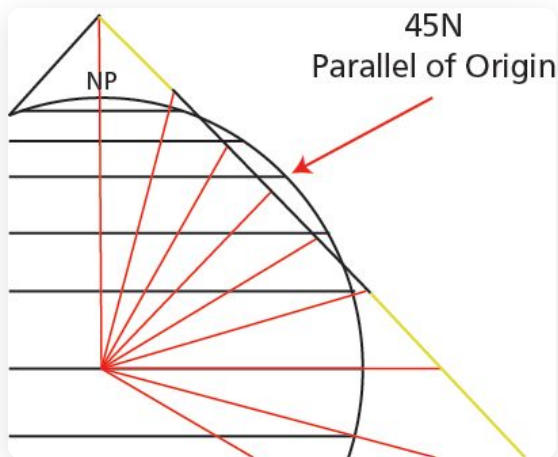


Figure 11 — Lambert scale zones: scale contracts between the standard parallels (blue, scale < 1), is correct AT the standard parallels (green), and expands outside (yellow/red)

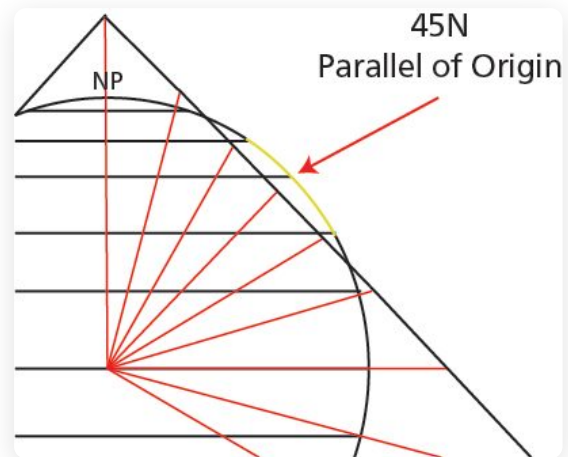


Figure 12 — Lambert scale zones continued — the expanded and contracted zones are exaggerated here to show the principle; in practice the variation is tiny across the chart

Non-perspective Chart

Having brought the cone inside the Reduced Earth, Lambert made mathematical adjustments to ensure the chart is **orthomorphic**. The result is that the Lambert chart is a **non-perspective chart** — it cannot be produced by a simple optical projection. It is constructed mathematically.

This is the same situation as the Mercator: the mathematical modification required to achieve conformality removes the direct optical projection relationship.

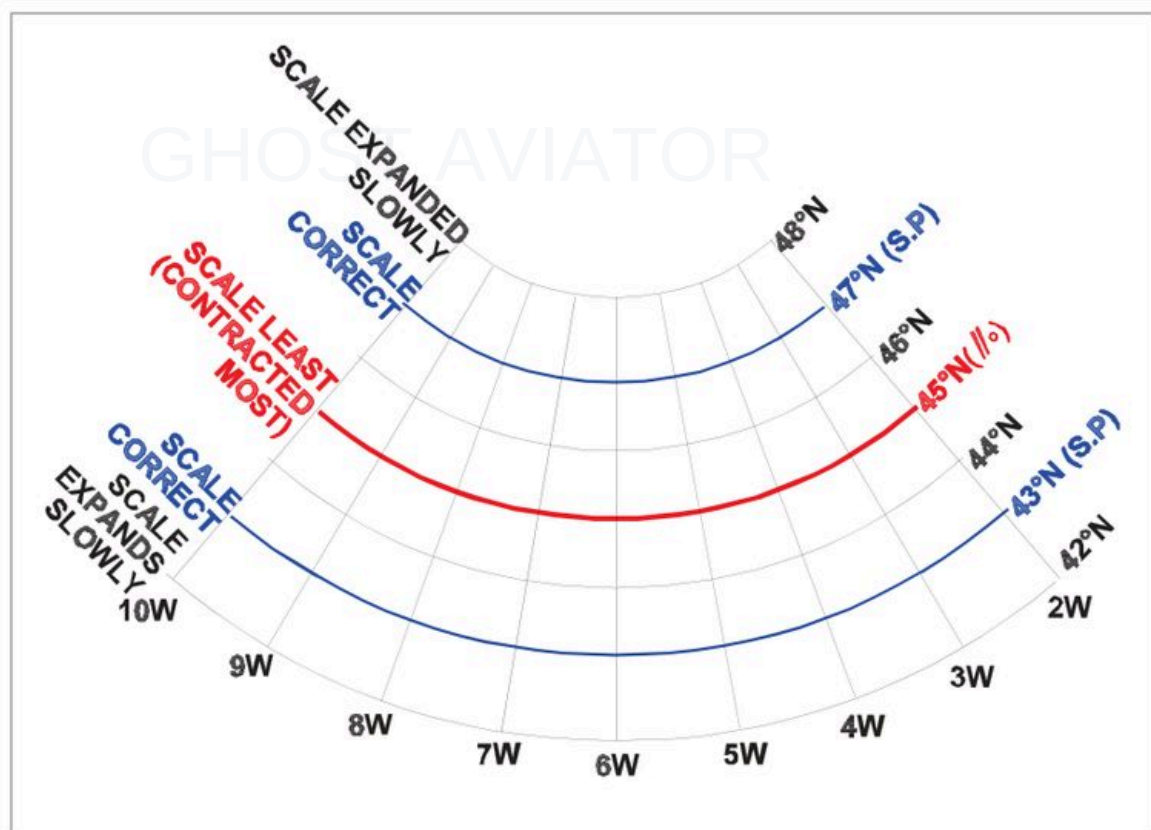


Figure 13 — Part of a Lambert conical orthomorphic chart — the familiar look of the ICAO 1:500 000 or 1:250 000 aeronautical chart; note slightly curved parallels and converging meridians

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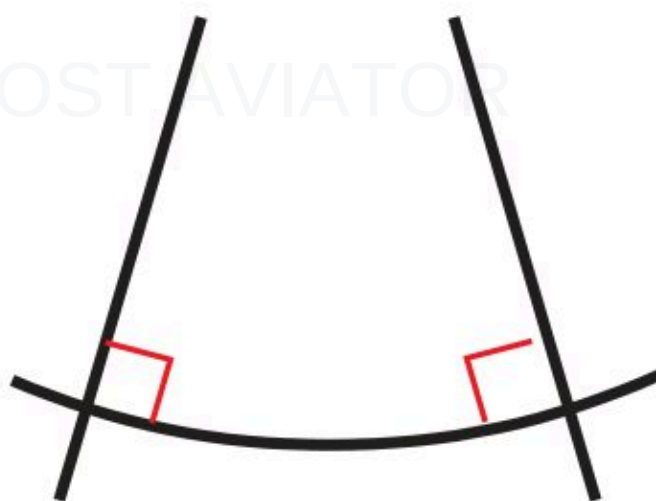


Figure 14 — *Orthomorphism on the Lambert chart: meridians converge (left) and parallels curve (right), but when combined they intersect at exactly 90° — satisfying both orthomorphism*

Lambert Chart Properties

Scale

Scale is **least at the Parallel of Origin** (between the two standard parallels). It increases outward, reaching the correct value at the two **standard parallels**. Scale is **greatest at the top and bottom parallels** of the projection.

Parallel of Origin

The **Parallel of Origin** is the mathematical basis of the projection. It is assumed to be halfway between the two standard parallels. Its sine is the **constant of the cone (n)**:

$$n = \sin(\text{Parallel of Origin})$$

The parallel of origin defines chart convergence and is approximately the mid-parallel of the chart

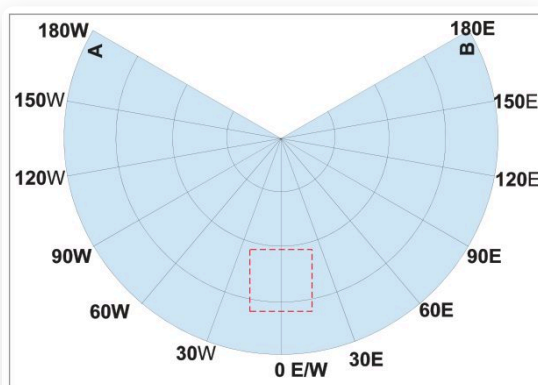


Figure 31.13

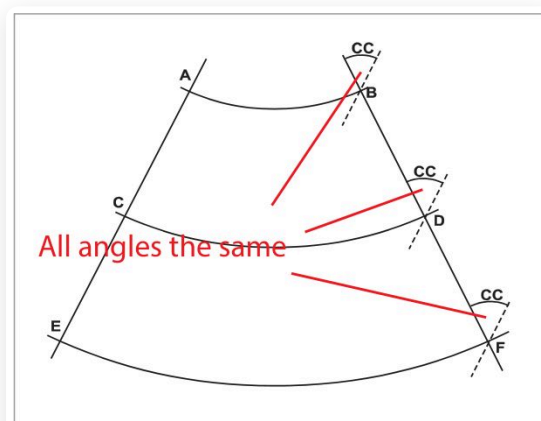


Figure 31.14 Chart convergence

Figure 15 — *The Parallel of Origin — the mathematical basis of the Lambert projection; it lies halfway between the two standard parallels and defines the convergence factor n*

Figure 16 — *Chart convergence on the Lambert chart is constant across the whole chart — all meridians incline at the same angle to each other*

Graticule

- **Meridians:** Straight lines radiating from the pole (pole is usually off the map)
- **Parallels:** Arcs of concentric circles centred at the pole
- Meridians and parallels intersect at 90°

Chart Convergence

Since the meridians on the Lambert chart are **straight lines**, the angle between any two given meridians does *not* change with latitude — unlike on the real Earth where meridians converge increasingly toward the poles.

Chart Convergence = $ch.long \times \sin(\text{Parallel of Origin})$

This is constant everywhere on the Lambert chart for any given $ch.long$

This is different from Earth convergency which = $ch.long \times \sin(\text{latitude})$ and changes with latitude. Chart convergence on the Lambert is *correct only at the Parallel of Origin*. Above the Parallel of Origin, chart convergence < Earth convergency; below it, chart convergence > Earth convergency.

Rhumb Lines and Great Circles

Rhumb Lines

Except for meridians (which are straight lines), rhumb lines on a Lambert chart appear as **curves concave to the pole** (i.e. they curve in the same direction as the parallels of latitude). The curvature is considerable.

Great Circles

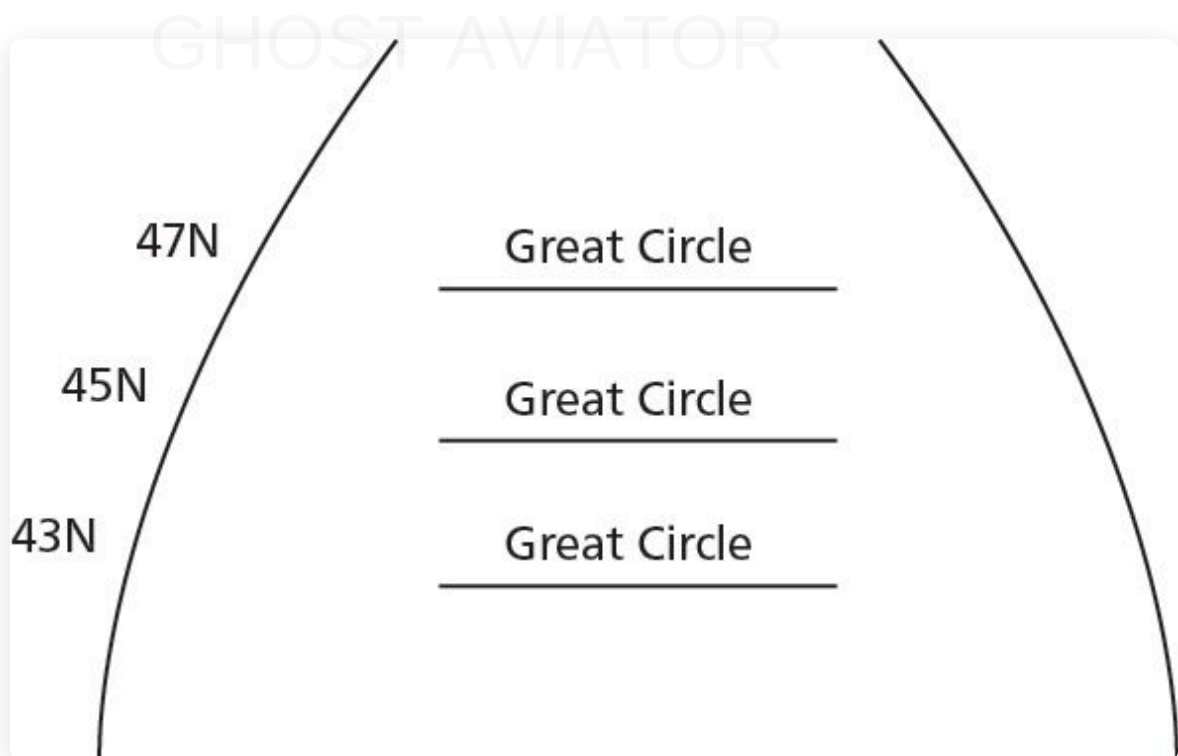


Figure 17 — Great circles on the real Earth — three Great Circle tracks at different latitudes, shown as straight lines across the globe

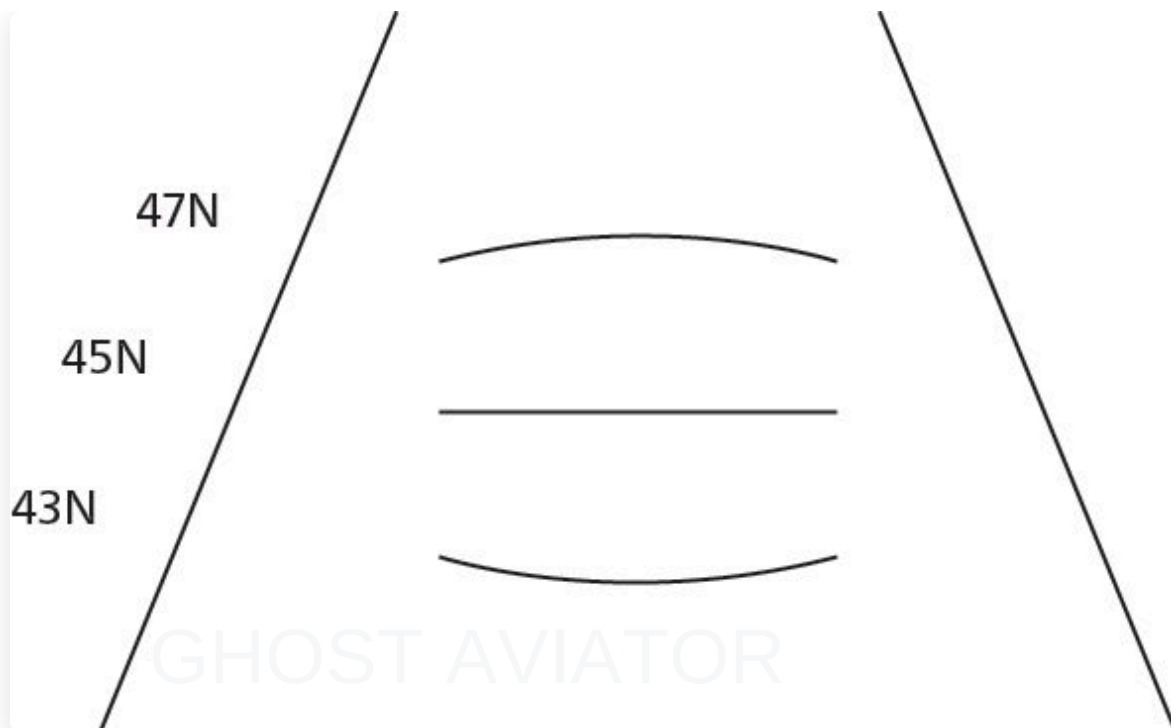


Figure 18 — *Great circles on a Lambert chart — when Earth meridians are straightened to parallel-line chart meridians, the formerly straight GCs become slightly curved; the curvature is concave to the Parallel of Origin*

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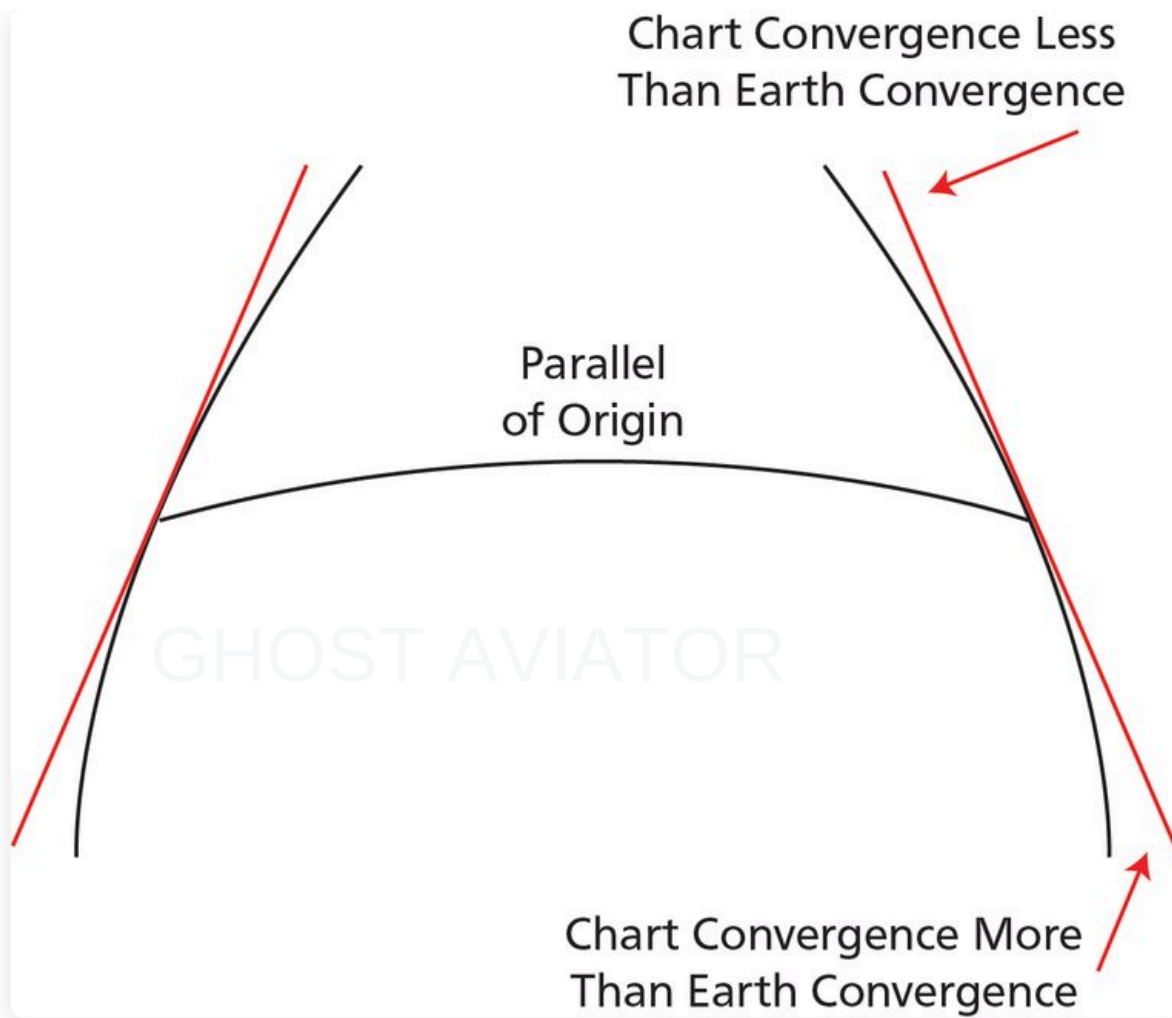


Figure 19 — Earth convergence vs chart convergence — the Lambert chart convergence is fixed at the Parallel of Origin value; at the Parallel of Origin, chart = Earth, and a Great Circle is a straight line

THE KEY RULE FOR GREAT CIRCLES ON LAMBERT CHARTS

A Great Circle is a straight line *only* at the Parallel of Origin. Elsewhere it curves concave to the Parallel of Origin. However, the amount of curvature is very small — much less than a Rhumb Line — and for **all practical purposes including plotting, Great Circles on a Lambert chart may be treated as straight lines.**

Summary — Properties of a Lambert Conformal Chart

Projection type	Conical, non-perspective, orthomorphic (conformal)
Scale	Correct at the two standard parallels. Contracted between them (least at Parallel of Origin). Expanded outside them.
Graticule	Meridians: straight lines from pole. Parallels: arcs of circles centred at pole.
Parallel of Origin	Mathematical basis of projection. $n = \sin(\text{PO})$. Halfway between standard parallels.
Chart Convergence	Constant: $\text{CC} = \text{ch.long} \times \sin(\text{Parallel of Origin})$. Correct only at PO.
Rhumb Lines	Meridians = straight. All other RLs = curves concave to the pole.
Great Circles	Meridians = straight. At PO = near-straight. Elsewhere = curves concave to PO (but very slight). Treated as straight lines for plotting.

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